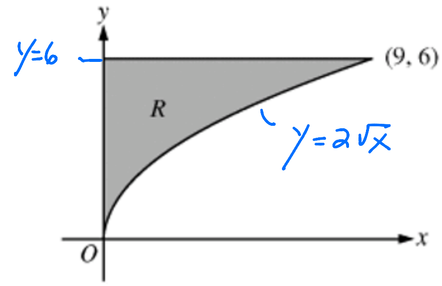
Period 3, April 21, 2025



(a) $\int_0^9 (6-2\sqrt{x})\,dx$

$6x - 2\cdot\frac{2}{3}x^{\frac{1}{2}+1=\frac{3}{2}} + C \Big|_0^9$

$6\cdot 9 - \frac{4}{3}(9)^{3/2} - \cancel{\left(6\cdot 0 - \frac{4}{3}0^{3/2}\right)}$ 0

$54 - 4\cdot 9 = 54 - 36$

18

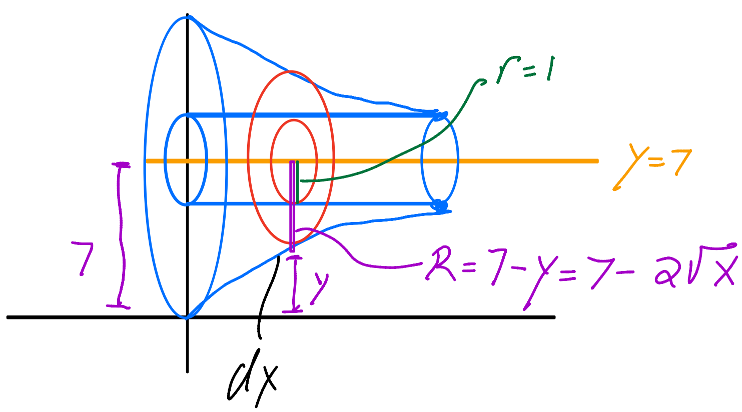
4. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

   (c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

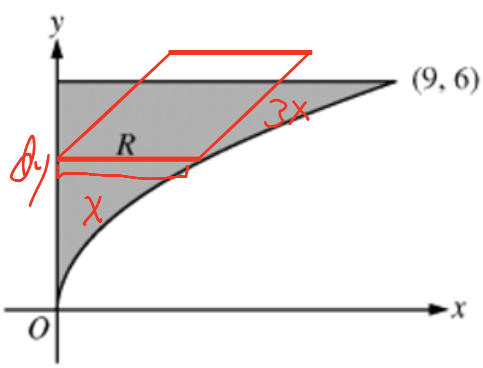
(b)



$\pi\int (R^2 - r^2)\,dx$

$\pi\int_0^9 \left[(7-2\sqrt{x})^2 - (1)^2\right]dx$

(c)



$\int_0^6 x\cdot 3x\cdot dy$

$\int_0^6 \frac{y^2}{4}\cdot 3\cdot\frac{y^2}{4}\,dy$

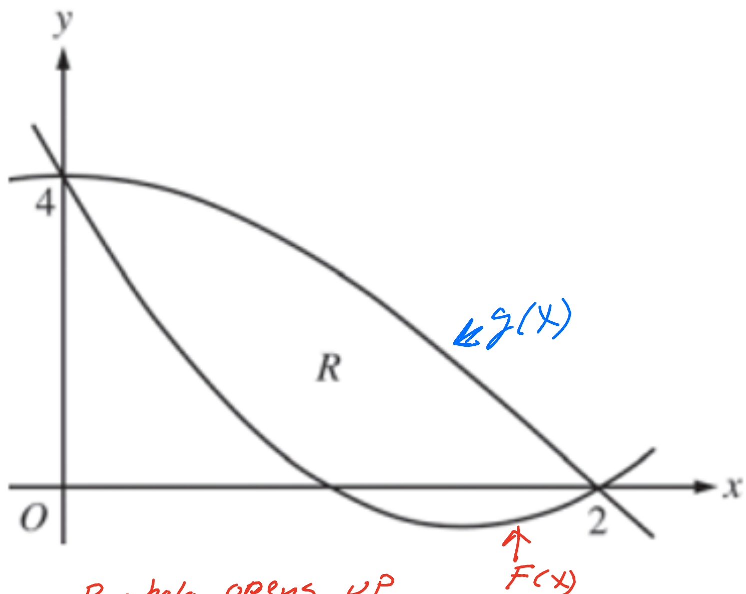
$\int_0^6 \frac{3y^4}{16}\,dy$

$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$\left(\frac{y}{2}\right)^2 = x$

$\frac{y^2}{4}$



Parabola opens up

5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R. $\int_0^2 (g(x) - F(x))\,dx$

$$\int_0^2 \left(4\cos\frac{\pi x}{4} - (2x^2 - 6x + 4)\right)dx$$

$$\int_0^2 \left(4\cos\frac{\pi x}{4} - 2x^2 + 6x - 4\right)dx$$

$$\frac{16}{\pi}\sin\frac{\pi x}{4} - 2\cdot\frac{1}{3}\cdot x^{\overset{2+1=3}{}} + 6\cdot\frac{1}{2}\cdot x^{\overset{1+1=2}{}} - 4x\Big|_0^2$$

$$\frac{16}{\pi}\sin\underbrace{\frac{2\pi}{4}}_{\frac{\pi}{2}} - \frac{2}{3}(2)^3 + 3\cdot 2^2 - 4(2) - \cancel{\left[\frac{16}{\pi}\sin 0 - 0 + 3\cdot 0 - 0\right]}$$

$$\frac{16}{\pi}\cdot 1 - \frac{16}{3} + 12 - 8 = \frac{16}{\pi} - \frac{16}{3} + 4 = \frac{16}{\pi} - \frac{16}{3} + \frac{12}{3}$$

$$\frac{16}{\pi} - \frac{4}{3}$$

$$\int 4\cos\frac{\pi x}{4}$$

$$u = \frac{\pi x}{4}$$

$$\frac{4}{\pi}\cdot du = \cancel{\frac{\pi}{4}}\cdot dx\cdot\cancel{\frac{4}{\pi}}$$

$$\frac{4}{\pi}du = dx$$

$$\int 4\cdot\cos u\cdot\frac{4}{\pi}du$$

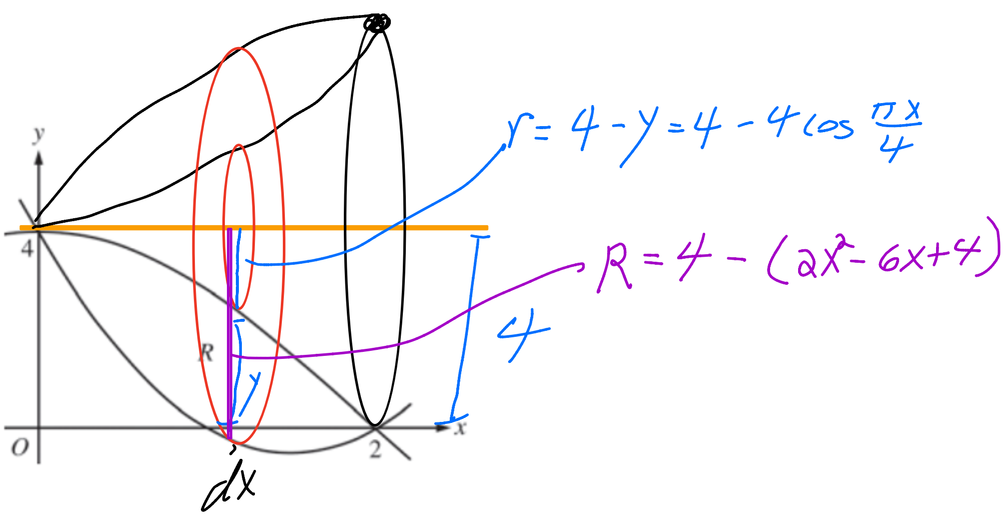
$$\frac{16}{\pi}\int\cos u\,du$$

$$\frac{16}{\pi}\sin u = \frac{16}{\pi}\sin\frac{\pi x}{4}$$

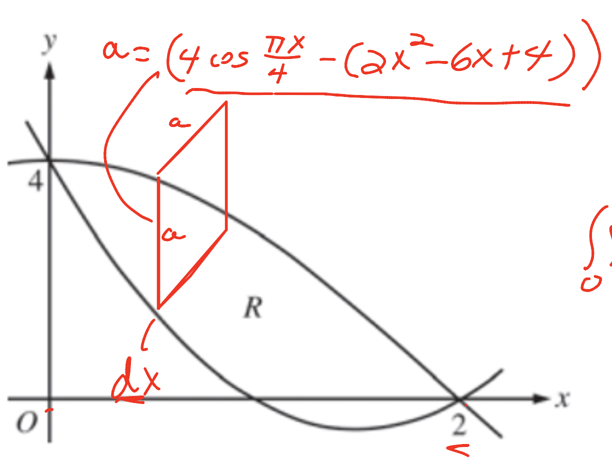
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

$$\pi\int_0^2 (R^2 - r^2)\,dx$$

$$\pi\int_0^2 \left[\left(4-(2x^2-6x+4)\right)^2 - \left(4-4\cos\frac{\pi x}{4}\right)^2\right]dx$$



(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.



area of Square

$$\int_0^2 (a)^2\,dx$$

$$\int_0^2 \left[4\cos\frac{\pi x}{4} - (2x^2-6x+4)\right]^2 dx$$

$$\int_0^2 \left[4\cos\frac{\pi x}{4} - 2x^2 + 6x - 4\right]^2 dx$$

2. (1985 BC3-appropriate for AB)

$$\int_1^2 \frac{x+1}{x^2+2x}\,dx =$$

$u = x^2+2x$

$du = (2x+2)\,dx \Rightarrow \frac{du}{2x+2} = dx \Rightarrow \frac{du}{2(x+1)} = dx$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

$\int \frac{\cancel{x+1}}{u} \cdot \frac{du}{2\cancel{(x+1)}} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+2x| + C \Big|_1^2$

$\frac{1}{2}\ln|\overset{\overset{8}{4+4}}{2^2+2\cdot 2}| - \frac{1}{2}\ln|\overset{\overset{3}{1+2}}{1^2+2\cdot 1}|$

$\frac{1}{2}\ln 8 - \frac{1}{2}\ln 3 = \frac{\ln 8 - \ln 3}{2}$

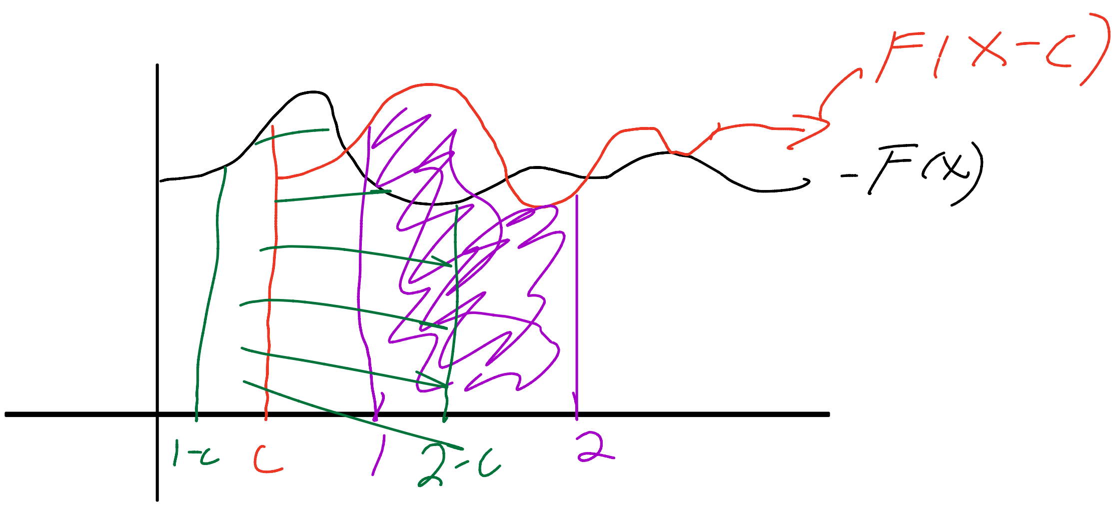
4. (1973 BC38-appropriate for AB)

moves F(x) Right C

move interval Right C

If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) -5



3. (1988 AB10)

$$\cancel{2}k \cdot \frac{1}{\cancel{2}} \cdot x^{1+1=2} - \frac{1}{3}x^{2+1=3} \Big[_0^k$$

$$k \cdot k^2 - \frac{1}{3} \cdot k^3 - \left[\cancel{k \cdot 0^2 - \frac{1}{3}(0)^3}\right]$$

$$k^3 - \frac{1}{3}k^3 = \frac{2}{3}k^3$$

If $\int_0^k \left(2kx - x^2\right) dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

$$\cancel{\frac{3}{2} \cdot \frac{2}{3}} k^3 = 18 \cdot \frac{3}{2}$$

$$k^3 = 9 \cdot 3 = 27$$

$$k^3 = 27 \qquad k = 3$$

$$u = \sqrt{x} = x^{\frac{1}{2}}$$

$$du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}dx$$

$$2\sqrt{x} \cdot du = dx$$

Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\, du$ (B) $2\int_1^4 e^u\, du$ (C) $2\int_1^2 e^u\, du$ (D) $\frac{1}{2}\int_1^2 e^u\, du$ (E) $\int_1^4 e^u\, du$

$$\int_1^2 \frac{e^u}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\, du = 2\int_1^2 e^u\, du$$

$$u = \sqrt{x}$$

$$x = 4$$

$$u = \sqrt{4} = 2$$

$$x = 1$$

$$u = \sqrt{1} = 1$$

10. (1969 AB4)

$$\int_0^8 \frac{dx}{\sqrt{1+x}} =$$

$$u = 1 + x$$

$$du = dx$$

$$\int \frac{dx}{\sqrt{u}} = \int \frac{du}{u^{\frac{1}{2}}} = \int u^{-\frac{1}{2}}du = \frac{2}{1}u^{-\frac{1}{2}+1=\frac{1}{2}} + C$$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

$$2\sqrt{u} = 2\sqrt{1+x}\Big[_0^8 = 2\sqrt{1+8} - 2\sqrt{1+0}$$

$$2\sqrt{9} - 2\sqrt{1} = 2 \cdot 3 - 2 \cdot 1 = 6 - 2 = 4$$

6. (1973 AB30)

$\int_1^2 \frac{x-4}{x^2}\,dx =$ $\int_1^2 \left[\frac{x}{x^2} - \frac{4}{x^2}\right] dx = \int_1^2 \left(\frac{1}{x} - 4\cdot x^{-2}\right) dx = \ln|x| - 4\cdot\frac{1}{-1}\cdot x^{-2+1=-1}$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

$\ln|x| + 4\cdot x^{-1} = \ln x + \frac{4}{x}\Big]_1^2$

$\ln 2 + \frac{4}{2} - \left[\ln|1| + \frac{4}{1}\right]$

$\ln 2 + 2 - 0 - 4 = \ln 2 - 2$

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7. (2012 AB3)

$\int \sec x \tan x \, dx =$

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\frac{\sec^2 x}{2} + C$

(D) $\frac{\tan^2 x}{2} + C$

(E) $\frac{\sec^2 x \tan^2 x}{2} + C$

$U = \sec x$

$dU = \sec x \tan x \, dx$

$\frac{dU}{\sec x \tan x} = dx$

$\int \frac{\cancel{\sec x \tan x} \, dU}{\cancel{\sec x \tan x}}$

$\int 1 \, du = u + c = \sec x + C$

12. (1969 AB38)

$u = x^3$

$du = 3x^2\,dx \Rightarrow \frac{du}{3x^2} = dx$

$$\int \frac{x^2}{e^{x^3}}\,dx =$$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

$$\int \frac{\cancel{x^2}}{e^u} \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int \frac{du}{e^u} = \frac{1}{3}\int e^{-u}\,du = \frac{1}{3}\int e^{L} \cdot -dL$$

$L = -u$

$dL = -du$

$-dL = du$

$-\frac{1}{3}\int e^{L}\,dL$

$-\frac{1}{3}e^{L} = -\frac{1}{3}e^{-u}$

$= -\frac{1}{3}e^{-x^3} = \frac{1}{-3e^{x^3}}$

14. (1997 AB3)

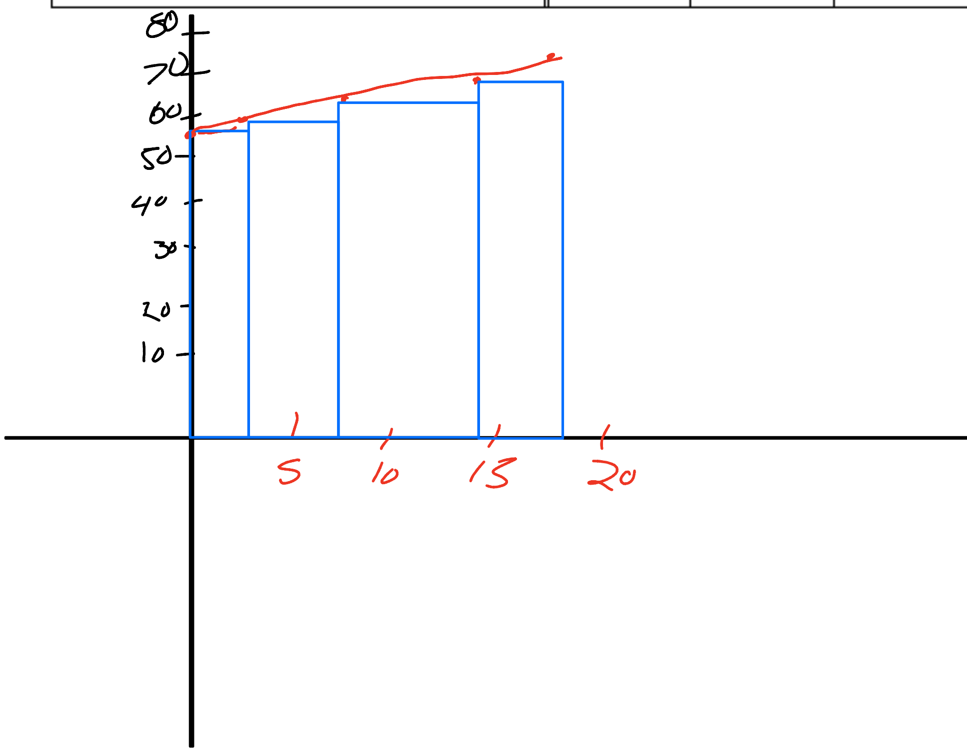
If $\int_a^b f(x)\,dx = \underline{a + 2b}$, then $\int_a^b (f(x) + 5)\,dx =$

(A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

$$\int_a^b (F(x) + 5)\,dx = \underline{\int_a^b F(x)\,dx} + \int_a^b 5\,dx = a + 2b + 5b - 5a = 7b - 4a$$

$5x \Big|_a^b = \underline{5b - 5a}$

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0



(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

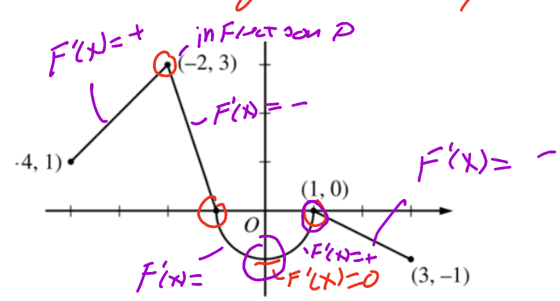
change $g''(x)$ signs

$g''(x) = 0$ or $\emptyset$

$g(x) = \int^{x} F(T)\,dT$

$g'(x) = F(x)$

$g''(x) = F'(x) =$ Slope of $F(x)$



Graph of f

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dT}{100-B} \cdot \frac{dB}{dt} = \frac{1}{5}(100 - B) \cdot \frac{dT}{100-B} = \text{Rate of weight gain}$$

$$\int \frac{1}{100-B} \cdot dB = \int \frac{1}{5} dT$$

$U = 100 - B$

$dU = -dB$

$-dU = db$

$$\int \frac{1}{U} \cdot -dU$$

$-\ln|U|$

$-\ln|100-B| = \frac{1}{5}T + C$

$$= \frac{dB}{dT} = \frac{1}{5}(100 - B)$$

greater rate

$\frac{1}{5}(100-40) = \frac{60}{5} = 12$

$\frac{1}{5}(100-70) = \frac{30}{5} = 6$

(b) $\frac{dB}{dT} = \frac{1}{5}(100-B) = 20 - \frac{1}{5}B$

$$\frac{d^2B}{dT^2} = 0 - \frac{1}{5}\frac{dB}{dT} = -\frac{1}{5}\left(\frac{1}{5}(100-B)\right) = -\frac{1}{25}(100-B)$$

Concave Down until B = 100